

Are you Ready for AP Calculus?

Summer Packet

This is due the first day of school. You must do these problems neatly, legibly, and in order on a separate piece of paper. You must show work leading to your answer. Use graph paper to create scaled and presentable graphs without the use of a calculator. Make sure your graphs show the significant details of each specific function. There is a key at the end to help you, you are responsible for knowing the packet material.

Simplify.

1. $\frac{x+5}{x^2-2x-35}$ 2. $\frac{x^3-64}{x-4}$ 3. $\frac{7-x}{x^2-49}$ 4. $\frac{16x+3x^2-35}{33x+5x^2-14}$

Complete the following with an alternate value.

5. $\sin^2 x + \cos^2 x =$ 6. $1 + \tan^2 x =$ 7. $\cot^2 x + 1 =$
8. $\sin(2x) =$ 9. $\cos(2x) =$ 10. $\sin(x + y) =$
11. $\cos(x - y) =$ 12. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ 13. $\cos^{-1}\left(-\frac{1}{2}\right) =$
14. $\tan^{-1}(0) =$

Simplify.

15. $\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$ 16. $\frac{\frac{x^2}{y^2} - 1}{\frac{x^2}{y^2} - \frac{2x}{y} + 1}$ 17. $\frac{x-y}{xy} + \frac{x-z}{xz} - \frac{z-y}{yz}$
18. $\frac{45a^3b^2}{28c^4d^3} \div \frac{-75a^4b}{8c^2d^4}$

If $f(x) = \{(2, 3), (4, 5), (7, 2)\}$; $h(x) = \{(2, 1), (3, 2), (7, 4)\}$; $g(x) = 3x^2 + 1$, if $0 \leq x$; $k(x) = \sqrt{x + 2}$ then determine each of the following:

19. $(f + h)(2) =$ 20. $(k - g)(2) =$ 21. $(f \circ h)(7) =$
22. $(g \circ k)(7) =$ 23. $f^{-1}(x) =$ 24. $g^{-1}(x) =$
25. $\frac{f(4)}{g(2)} =$ 26. $(gk)(x) =$
27. Use $f(x) = x^2 + 2x$, to evaluate $\frac{f(x+h)-f(x)}{h}$ and simplify your answer.

Use the limit definition of a derivative: $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative of each function at the given x value.

28. $f(x) = 4 - \sqrt{x + 3}$ at $x = 2$ 29. $f(x) = \frac{x+1}{2-x}$ at $x = 3$
30. $f(x) = 5x^2 + 3x - 7$ at $x = -2$

Simplify. (continued on next page)

31. $x^{\frac{1}{2}}\left(\sqrt[3]{x^2} + x^2 - x\right)$ 32. $e^{(2+\ln x)}$ 33. $e^{\ln 3}$
34. $\ln(e^2)$ 35. $e^{\frac{1}{2}\ln x}$ 36. $\log_4 \frac{1}{4}$

37. $\log_{0.5} 8$

38. $9^{\frac{1}{2}}$

39. $\log_4 9^2 - \log_4 3$

40. $\frac{4xy^{-2}}{6x^4y^{-3}}$

41. $\left(5a^{\frac{4}{3}}\right)\left(2a^{\frac{1}{3}}\right)$

42. $\left(4a^{\frac{7}{3}}\right)^{\frac{3}{2}}$

43. **Without a calculator**a. Graph $y = \sin x$ and $y = \cos x$ on $[0, 2\pi]$ using two different colors on the same coordinate plane.

b. Determine the x intercepts for each graph.

c. Determine the points of intersection of the two graphs.

Without a calculator, on graph paper, graph the functions. The answer the following questions for each:

a. Even, odd, neither

b. Domain

c. Range

d. x-intercept(s)

e. Y-intercept

f. asymptotes

44. $y = -\frac{1}{2}x + 4$

45. $y = \sqrt{x - 4}$

46. $y = \sqrt{9 - x^2}$

47. $y = (x - 3)^2 - 4$

48. $y = -x^2 - 4x + 8$

49. $y = \frac{1}{x}$

50. $y = \frac{1}{x^2}$

51. $y = \frac{2}{x-4}$

52. $y = \sqrt[3]{x}$

53. $y = -|x - 4| + 2$

54. $y = \ln(x)$

55. $y = e^x$

56. Show that $\frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta}$ can be simplified to $\csc \theta$.

57. Rewrite $\frac{1}{1 + \sin \theta}$ so that it is not in fractional form.

58. Show that $\frac{\sec^2 \theta - 1}{\sec^2 \theta}$ can be simplified to $\sin^2 \theta$.

59. Car A travels 20 miles per hour faster than car B. Car A can travel 100 miles in the same time it takes car B to travel 60 miles. Find the speed of each car.

60. A group of students wish to split the \$100 cost of reserving a room for a party. When 5 more students join the group, the cost per student is reduced by \$10. How many students were in the original group? What was the original cost per student?

61. An open box is made from a 10-inch piece of square cardboard by cutting a square from each corner and folding up the edges. If the area of the bottom of the box is 60 square inches, what is the length of the sides of the squares removed?

62. The speed of a boat in still water is 10 miles per hour. It travels 15 miles upstream and 15 miles downstream in a total time of 4 hours. What is the speed of the current?

63. A rectangular garden is 12 feet long and 10 feet wide. Part of the garden is torn up to install a sidewalk of uniform width around the garden. If the area of the new garden is 48 square feet, find the width of the sidewalk.

Answer Key - To check against - show your work

- ① $\frac{1}{x-7} \quad x \neq 7$
 ③ $-\frac{1}{x+7} \quad x \neq -7$
 ⑤ 1
 ⑦ $\csc^2 x$
 ⑨ $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
 ⑩ $\sin x \cos y + \cos x \sin y$
 ⑫ $-\frac{\pi}{2}$
 ⑭ 0
 ⑮ $\frac{x+y}{x-y} \quad x \neq -y \quad y \neq 0$
 ⑰ $-6bd/33ac^2$
 ⑲ -11
 ⑳ 28
 ㉑ $g^{-1}(x) = \sqrt{\frac{x-1}{3}}$
 ㉓ $g(x) = (3x^2+1)\sqrt{x+2} \quad x \geq -2$
 ㉕ $h+2x+2$
 ㉗ 1
 ㉙ $2x - |x|\sqrt{x}$
 ㉛ 3
 ㉝ $\sqrt{x}$
 ㉞ -3
 ㉟ $\log_4(27)$
 ㊱ $10a^{5/3} = 10^3 \sqrt[3]{a^5}$
 ㊲ @ see graph paper
- ② $x^2+4x+16 \quad x \neq 4$
 ④ $\frac{3x-5}{x-2} \quad x \neq 2$
 ⑥ $\sec^2 x$
 ⑧ $2\sin x \cos x$
 ⑩ $\cos x \cos y + \sin x \sin y$
 ⑫ $2\pi/3$
 ⑭ $\frac{y-x}{y+x} \quad x \neq y \quad x \neq 0 \quad y \neq 0$
 ⑮ $\frac{2(x-z)}{x-z} \quad x \neq 0 \quad y \neq 0 \quad z \neq 0$
 ⑰ 4
 ⑱ 5
 ㉑ $\{(3,2), (5,4), (2,7)\}$
 ㉓ $9/13$
 ㉕ $\frac{\sqrt{8}}{10}$
 ㉗ 23
 ㉙ $x e^2$
 ㉛ 2
 ㉝ -1
 ㉞ 3
 ㉟ $\frac{24}{3}$
 ㊱ $8a^{7/2} = 8\sqrt{a^7}$

* ④ a neither

- b $(-\infty, \infty)$
 c $(-\infty, \infty)$
 d $(8, 0)$
 e $(0, 4)$

④ a neither

- b $[0, \infty)$
 c $[0, \infty)$
 d $(0, 0)$
 e $(0, 0)$

④ a even

- b $[-3, 3]$
 c $[0, 3]$
 d $(-3, 0) \cup (3, 0)$
 e $(0, 3)$
 f none

④ a Neither

- b $(-\infty, \infty)$
 c $(-4, \infty)$
 d $(1, 0) \cup (5, 0)$
 e $(0, 5)$
 f none

④ a neither

- b $(-\infty, \infty)$
 c $(-\infty, 12]$
 d $(-2+2\sqrt{3}, 0) \cup (2-2\sqrt{3}, 0)$
 e $(0, 8)$
 f none

④ a odd

- b $(-\infty, 0) \cup (0, \infty)$
 c $(-\infty, 0) \cup (0, \infty)$
 d none
 e none
 f $x=0, y=0$

⑤ a even

- b $(-\infty, 0) \cup (0, \infty)$
 c $(0, \infty)$
 d none
 e none
 f $x=0, y=0$

⑤ a neither

- b $(-\infty, 4) \cup (4, \infty)$
 c $(-\infty, 0) \cup (0, \infty)$
 d none
 e $(0, -1/2)$
 f $x=4, y=0$

⑤ a odd

- b $(-\infty, \infty)$
 c $(-\infty, \infty)$
 d $(0, 0)$
 e $(0, 0)$
 f none

⑤ a neither

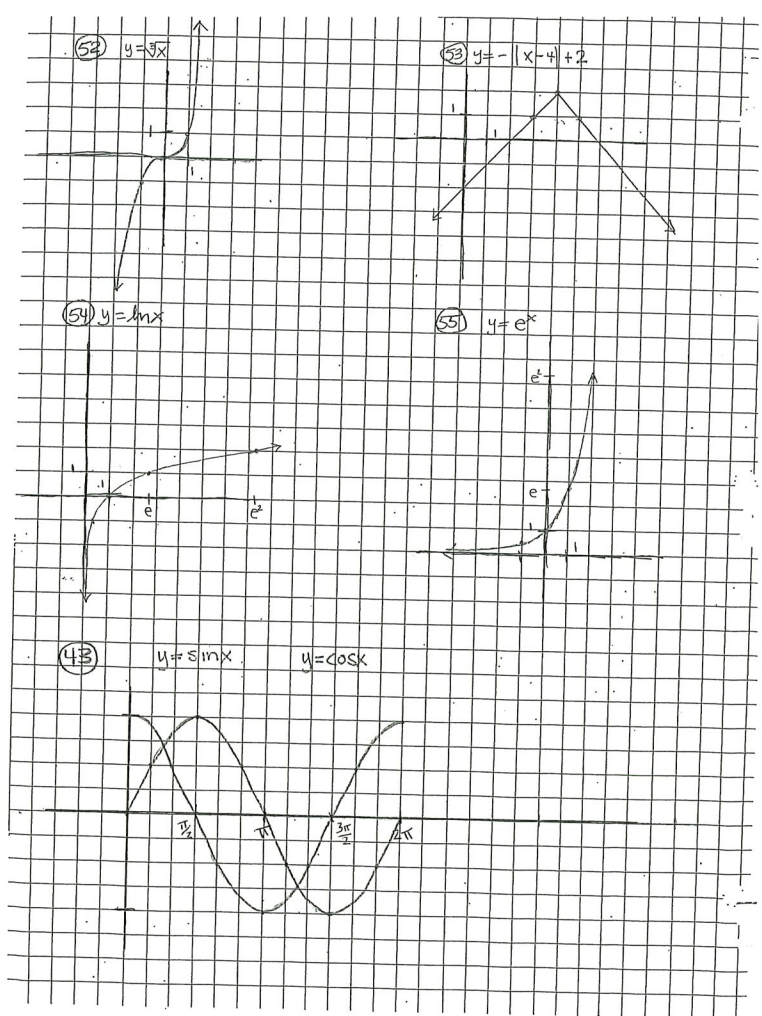
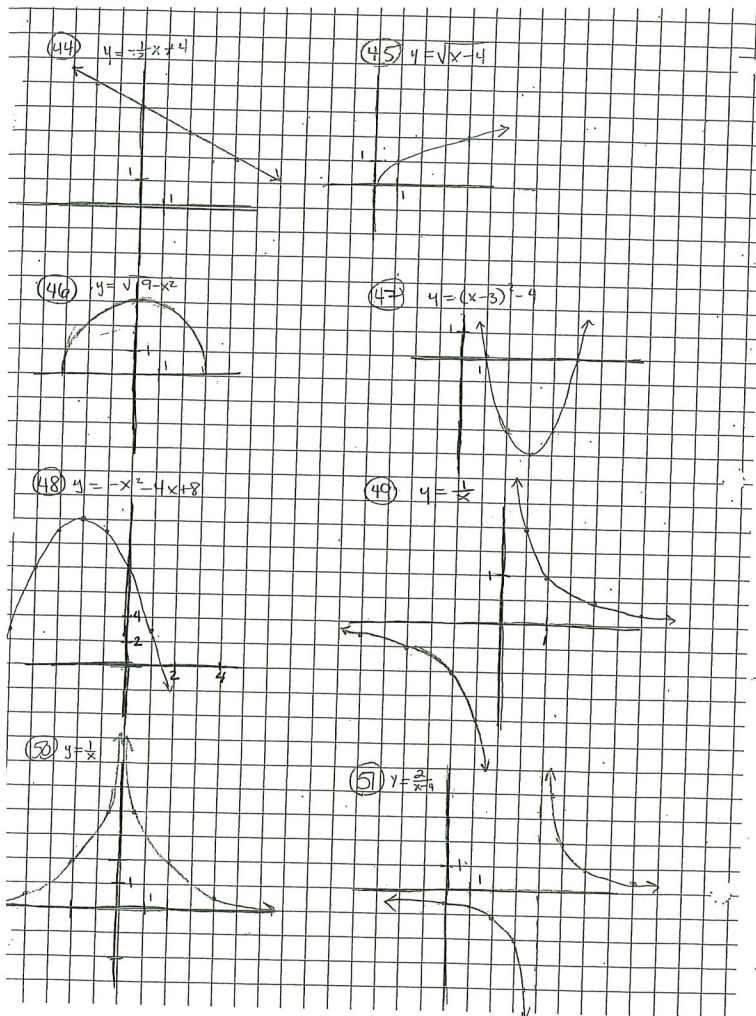
- b $(-\infty, \infty)$
 c $(-\infty, 2]$
 d $(2, 0)$ and $(6, 0)$
 e $(0, -2)$
 f none

⑤ a neither

- b $(0, \infty)$
 c $(-\infty, \infty)$
 d $(1, 0)$
 e none
 f $y=0$

⑤ a neither

- b $(-\infty, \infty)$
 c $(0, \infty)$
 d none
 e $(0, 1)$
 f $x=0$



56 - 58 answers will vary

59 Car A travels at 50mph
Car B travels at 30mph

60 There were 5 students in the original group
They each would have originally paid \$20.00

61 the length of the sides of the removed squares
is $\sqrt{10}$ inches

62 The speed of the current is 5mph

63 The width of the sidewalk is 2 feet.